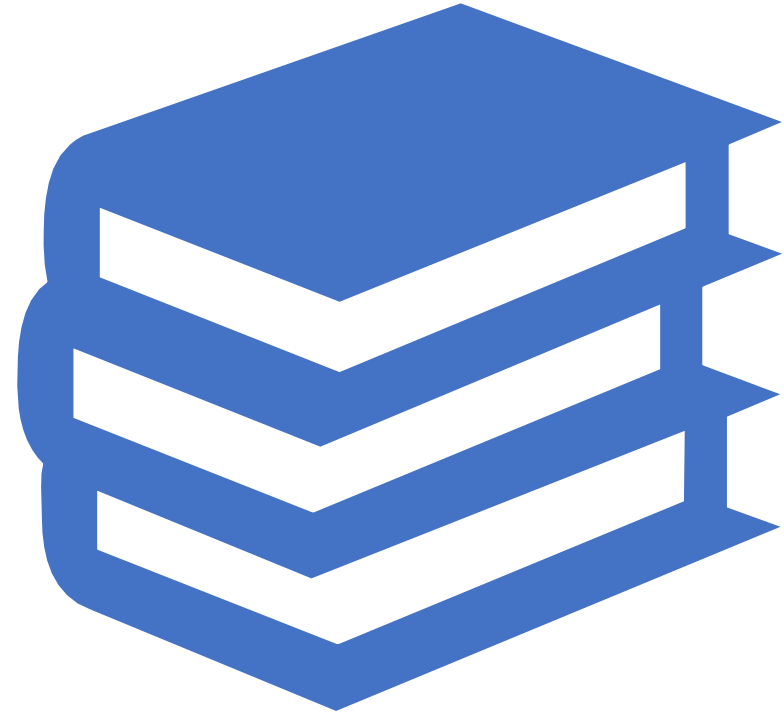


CS3243 Tutorial Group 7

Tutorial 3 – Informed Search



Quick recap

- Class of algorithms: best-first search
 - Use an evaluation function $f(n) = \text{cost estimate}$; expand node with the lowest estimate first
- Greedy best-first search
 - Evaluation function $f(n) = h(n)$
 - Heuristic function $h(n) = \text{estimated cost of cheapest path from } n \text{ to goal}$
 - Complete for graph-search (in finite spaces) but **not complete for tree-search** (even in finite spaces)
- A* search: avoid expanding paths that are already expensive
 - Evaluation function $f(n) = g(n) + h(n)$

- Heuristics:
 - Admissible (never overestimates)
 - Consistent/monotone = $f(n)$ is monotonically non-decreasing along every path
 - Dominance: dominant heuristics expand fewer nodes
 - Deriving admissible heuristics from relaxed problems
- Local search: hill-climbing

Intuition for admissible vs consistent

- **admissible** if it always underestimates path costs to the goal:

$$\forall n, \\ h(n) \leq h^*(n)$$

- **consistent** if it always underestimates step costs between nodes:

$$\forall n, n', \\ h(n) \leq d(n, n') + h(n') \quad \text{(original statement)}$$

Equivalently:

$$\underline{h(n) - h(n')} \leq d(n, n') \quad \text{(restatement)}$$

Now obvious why
consistent implies
admissible!

implied estimate of the
step cost between n and n'

Simple proof that UCS is optimal (step costs $\geq \epsilon$)

A with consistent $h(n)$ has the same proof*

- We use these invariants of UCS, which follow directly from the construction of the algorithm:

1. The frontier expands by exploring all edges out of the initial node, i.e. it forms a perimeter around it.
2. Since all step costs $\geq \epsilon$, $g(n)$ is **monotonically non-decreasing along any path.**
3. When a node n is chosen for expansion, $g(n)$ is \leq the cost of any other nodes in the frontier.

Consistency guarantees this; just replace $g(n)$ with $f(n)$ throughout

- Together, these invariants imply the property P:

whenever a node n is chosen for expansion, $g(n)$ is the shortest path cost from the initial node to n

- Proof (by contradiction):

- n is chosen for expansion.
- Suppose there exists a path from the initial node to n that has lower cost than $g(n)$.
- By inv. 1, the only way to find such a path is by expanding one of the nodes in the frontier (expanding the perimeter).
- But by inv. 3, all nodes q in the frontier have $g(q) \leq g(n)$, since n was selected for expansion.
- However, by inv 2., all paths from any node q have cost at least $g(q) + \epsilon$, which is strictly greater than $g(n)$.
Contradiction.

- By P, when a goal node is expanded, it has the lowest path cost

- i.e. UCS is optimal